Name..ID..............................Section.......1........Seat No............

Sirindhorn International Institute of Technology
Thammasat University

Midterm Examination: Semester 1 / 2018

Course Title: ECS315 (Probability and Random Processes)

Instructor: Asst. Prof. Dr.Prapun Suksompong

Date/Time: October 4, 2018 / 09:00 - 11:00

Instructions:

- This examination has.....8.....pages (including this cover page).
- Conditions of Examination:

☐ Open book			
☐ Closed book			
☑ Semi-Closed book (.......1.........sheet(s) ☑ 1 page ☐ both sides of A4 paper note) This sheet must be hand-written. They should be **<u>submitted with the exam</u>**. Do not modify (,e.g., add/underline/highlight) content on the sheet inside the exam room. Indicate your name and ID in the upper-right corner of the sheet (in portrait orientation). Other requirements are specified on the course website. (-10 pt if not following the requirements.)			
☐ Other:..			
☐ No dictionary	☐ Dictionary allowed	☐ No calculator	☑ Calculator allowed

- **<u>Read</u> these instructions and the questions carefully**.
- Students are not allowed to be out of the examination room during examination.
  Going to the restroom may result in score deduction.
- Turn off all communication devices and place them with other personal belongings in the area designated by the proctors or outside the test room.
- Write your name, student ID, section, and seat number clearly in the spaces provided on the top of this sheet.
  Then, write your **first name and the <u>last three digits</u> of your ID** in the spaces provided on the top of each page of your examination paper, starting from page 2.
- The examination paper is not allowed to be taken out of the examination room.
  Violation will result in a zero (0) score for the examination. Also, <u>do not remove the staple</u>.
- Unless instructed otherwise, ***<u>write down all the steps</u>*** that you have done to obtain your answers.
  - When applying formula(s), state clearly which formula(s) you are applying before plugging-in numerical values.
  - You may not get any credit even when your final answer is correct without showing how you get your answer.
  - Formula(s) not discussed in class can be used. However, derivation must also be provided.
  - **<u>Exceptions</u>**:
    - Problems that are labeled with "**ENRPr**" (Explanation is not required for this problem.)
    - Parts that are labeled with "**ENRPa**" (Explanation is not required for this part.)
    - These problems/parts are graded solely on your answers. There is no partial credit and it is not necessary to write down your explanation. Usually, spaces (boxes or cells in a table or rows of dashes) will be provided for your answers. "WACSP" stands for "write your answer(s) in the corresponding space(s) provided".
- <u>The back of each page</u> will <u>not</u> be graded; it can be used for calculations of problems that do not require explanation.
- When not explicitly stated/defined, all notations and definitions follow ones given in lecture.
- Some points are reserved for *accuracy* of the answers and also for reducing answers into their *simplest* forms. Watch out for roundoff error. Unless specified otherwise, the error in your final answer should not exceed 0.1%.
  - For counting problem, the answer should be reduced into just an integer.
    Exception: When the answer is more than 10^9, you may leave the answer in some form of simplified expression.
- Points marked with * indicate challenging problems.
- Do not cheat. Do not panic. **Allocate your time wisely.**
- Don't forget to submit your fist online self-evaluation form by the end of today.

Problem 1. (6 pt) [ENRPr] Let

$\mathbb{N}$ = the set of all natural numbers,
A = the interval $[0, 2]$,
B = the set of all real-valued x satisfying $\cos(x) = x^2 + 2$,
C = the set of all real-valued x satisfying $\cos(x) \geq 0.5$, and
D = the set of all real-valued x satisfying $\cos(x) = e^x$.

For each of the sets provided in the first column of the table below, indicate (by putting a Y(es) or an N(o) in the appropriate cells of the table) whether it is "finite", "infinite", "countably infinite", "uncountable".

Set	Finite	Infinite	Countably Infinite	Uncountable
$\mathbb{N}$				
$\left\{1, 2, \ldots, 10^{10}\right\}$				
A				
B				
C				
D				

Problem 2. (1* pt) Without the help of your calculator, <u>show</u> how to simplify the following expression analytically:

$$2 \times \binom{2018}{2} + 4 \times \binom{2018}{4} + 6 \times \binom{2018}{6} + \cdots + 2018 \times \binom{2018}{2018}.$$

Problem 3. (5 pt) [ENRPr]

(a) (2 pt) A fair coin is flipped 9 times. The results are: THTHTHTHT. Let A be the event that H occurs. Let $R(n)$ be the relative frequency of event A for the first n flips. Find $R(4)$ and $R(9)$.

$$R(4) = \underline{\hspace{3cm}} \text{ and } R(9) = \underline{\hspace{3cm}}$$

(b) (3 pt) Calculate the following quantities:

(i) (1 pt) $5!$

(ii) (1 pt) $\binom{8}{5}$

(iii) (1 pt) $\binom{8}{1,2,5}$

Problem 4. (4 pt) [ENRPr] Suppose we sample 5 objects from a collection of 8 distinct objects. Calculate the number of different possibilities when

(a) the sampling is unordered and performed without replacement

(b) the sampling is unordered and performed with replacement

(c) the sampling is ordered and performed without replacement

(d) the sampling is ordered and performed with replacement

Problem 5. ($1 \times 5 + 1^* + 1^* = 7$ pt)

(a) Calculate the number of different results when we permute the letters in each of the following collections:

(i) ABCD

(ii) ABBCDDD

(b) In the expansion of $(x + y)^{2018}$, find the coefficient of $x^{2016}y^2$.

(c) In the expansion of $(x + 2y)^{2018}$, find the coefficient of $x^{2016}y^2$.

(d) Find the number of solutions to $x_1 + x_2 + x_3 = 9$.
Assume all variables are **nonnegative** integers.

(e) Find the number of solutions to $x_1 + x_2 + x_3 + x_4 = 13$.
Assume all variables are **positive** integers.

(f) Find the number of solutions to $x_1 + x_2 + x_3 + x_4 + \cdots + x_{10} = 7$.
Assume each variable can only take the value 0 or 1.

Problem 6. (4 pt) Calculate the probability of each event below. Your answers should be of the form 0.XXXX.

(a) Obtaining at least one six in four tosses of a fair dice.

(b) Obtaining at least one triple six in 158 tosses of three fair dice.

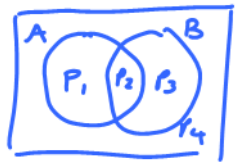
$P(R)$

Problem 7. (4 pt) Due to an Internet configuration error, packets sent from New York to Los Angeles are routed through El Paso with probability 1/5. Given that a packet is routed through El Paso, suppose it has conditional probability 1/4 of being dropped. Given that a packet is not routed through El Paso, suppose it has conditional probability 1/6 of being dropped. Find the conditional probability that a packet is routed through El Paso given that it is dropped.

D $\quad P(D|R) = \frac{1}{4}$ $\quad R$ $\quad P(D|R^c)$

$P(R|D)$

Problem 8. (1* pt) In the limit as $n \to \infty$, find the probability for exactly three occurrences of a 1-in-n-chance event in $2n$ repeated independent trials.



Problem 9. (1 pt) [ENRPr] For each of the following statement, write a T if it is always **true**. Write an F if the statement can be **false**; that is, write an F if one can find a counter-example to the statement.

F $P(A \cup B) = P(A) + P(B)$ $- P(A \cap B)$

T $P(A \cup B) - P(A \cup B^c) = P(A^c \cap B) - P(A^c \cap B^c)$

$(\cancel{p_1} + \cancel{p_2} + p_3) - (\cancel{p_1} + \cancel{p_2} + p_4) \quad p_3 - p_4$

Problem 10. (8 pt) [ENRPr] Consider a random experiment whose sample space is $\{a, b, c, d, e\}$ with probabilities 0.1, 0.2, 0.2, 0.2, 0.3, respectively. Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, and $C = \{c, d, e\}$.
Find the following probabilities.

$P(A) =$

$P(B) =$

$P(A \cap B) =$

$P((A \cup B) \cap C) =$

$P(A|B) =$

$P(A^c|B) =$

$P(A^c|B^c) =$

$P(A \cap C|B) =$

Problem 11. (8 pt) [ENRPr] In each of the parts below, find $P(A)$, $P(B)$, and $P(A \cap B)$. Then, determine (by putting a Y(es) or an N(o) in the provided space) whether events A and B are independent.

(a) $P(A^c \cap B^c) = 0.4$, $P(A \cap B^c) = 0.4$, and $P(A^c \cap B) = 0.1$.

$P(A) =$ _____, $P(B) =$ _____, $P(A \cap B) =$ _____; $A \perp\!\!\!\perp B$? _____

(b) $P(A \cup B) = 0.8$, $P(A \cup B^c) = 0.7$, $P(A^c \cup B) = 0.8$.

$P(A) =$ _____, $P(B) =$ _____, $P(A \cap B) =$ _____; $A \perp\!\!\!\perp B$? _____

$① \Leftrightarrow ②$

$① \Rightarrow ②$

$② \Rightarrow ①$

$P(A|B) = \frac{P(A\cap B)}{P(A)} \quad \frac{P(A\cap B)}{P(B)}$

$P(A)P(B) = P(A\cap B)$

Problem 12. (3 pt) [ENRPr] Suppose $P(A) > 0$, $P(B) > 0$, and $P(A\cap B) > 0$. For each of the given statements, indicate (by putting a Y(es) or an N(o) in the provided space) whether it is **equivalent** to the statement "A and B are independent".

N	$P(A\cap B) = P(B)P(A\vert B)$	Y	$P(A\cap B^c) = P(A)P(B^c)$
Y	$P(A\cap B) = P(A\vert B)P(B\vert A)$	Y	A^c and B^c are independent
?	$P(A\cup B) = P(A)P(B^c) + P(B)$	N	$P(A) = P(A\cap B) + P(A\cap B^c)$

Problem 13. (5 pt) Consider three events A,B, and C. Suppose

$$P(A^c\cap B\cap C) = \frac{1}{60}, \quad P(A\cap B^c\cap C) = \frac{7}{120}, \quad P(A\cap B\cap C^c) = \frac{1}{10},$$
$$P(A^c\cap B^c\cap C) = \frac{13}{120}, \quad P(A^c\cap B\cap C^c) = \frac{3}{20}, \quad P(A\cap B^c\cap C^c) = \frac{11}{40},$$
$$\text{and} \quad P(A\cap B\cap C) = \frac{1}{15}.$$

(a) (1 pt) Calculate the probability that exactly one of the three events occurs.

(b) (1 pt) Calculate the probability that none of the three events occurs.

(c) (1 pt) [ENRPa] Find $P((A\cup B)\cap(A^c\cup C))$.

(d) (1 pt) [ENRPa] Are A, B, and C pairwise independent?

(e) (1 pt) [ENRPa] Are A, B, and C independent?

Problem 14. ($2 \times 4 + 1 = 9$ pt) [Digital Communication] A certain binary-symmetric channel has a crossover probability (bit-error rate) of 0.35. Assume bit errors occur independently. Your answers should be of the form X.XXXX.

(a) Suppose we input bit sequence "00100" into this channel.

(i) What is the probability that exactly three bits are in error at the channel output ?

(ii) What is the probability that there is at least one bit error at the channel output?

(iii) What is the probability that the output is "10101"?

(b) Suppose we keep inputting bits into this channel. At the output, what is the probability that the *first* bit error occurs on the *third* bit?

(c) Suppose the transmitter uses repetition code (repeating each bit *three* times) and the receiver uses majority voting. Find the new (overall) error probability $P(\mathcal{E})$.

Problem 15. (1 pt)

(a) Do not forget to submit your study sheet with your exam.

(b) Make sure that you write your name and ID on every page. (Read the instruction on the cover page.)

(c) The online self-evaluation form is due by the end of today.